Schriftelijk Tentamen
"Cosmic Structure Formation"

MSc version
2^{nd} term, quarter Ib, 2016/2017

January 25, 2017

Note: 3 questions, 9 pages (incl. this frontpage) !
Please first browse through all questions.
Before you start answering a question, first read carefully the whole question (all items).

1 The galaxy mass function and the Schechter function

a) What is the definition of the mass function (MF) of galaxies? Give the definition in three ways: as a very short description that can convey the meaning of the MF to a non-expert in simple terms; as a strict definition in words; and in terms of a formula. What are the units of the MF according to the formula you have just written down?

Hint: The galaxy mass function is exactly the same as the galaxy luminosity function, but refers to galactic mass rather than galactic luminosity. In this context "mass" can refer to any component of a galaxy, such as stars ("stellar mass function"), atomic hydrogen gas ("HI mass function") or stars+atomic gas ("baryonic mass function").

b) We have seen in the lecture that the MF of galaxies measured in our universe can be described to fairly good accuracy by a specific functional form, called "Schechter function". If we denote the MF by $\phi(M)$, then the formula for the Schechter function is the following:

$$\phi(M)dM = \phi^* \left(\frac{M}{M^*}\right)^\alpha \exp\left(-M/M^*\right) d\left(\frac{M}{M^*}\right) \quad . \tag{1}$$

Can you describe in words the meaning of the three parameters (ϕ^*, M^*, α)? Can you give each parameters units?

c) Derive an expression for the mean space density $\langle n \rangle$ of galaxies, according to the Schechter function, i.e.

$$\langle n \rangle = \left\langle \frac{dN_{\rm gal}}{dV} \right\rangle \tag{2}$$

Hint: You may want to use the Gamma function in your expression:

$$\Gamma(z) \equiv \int_0^\infty t^{z-1} e^{-t} dt \tag{3}$$

d) Along the same line, derive an expression for the mean mass density $\langle M \rangle$ of galaxies according to the Schechter function.

e) Assume $\alpha < -1$. What would you conclude from the result of (c) and (d) ?

cont'd next page

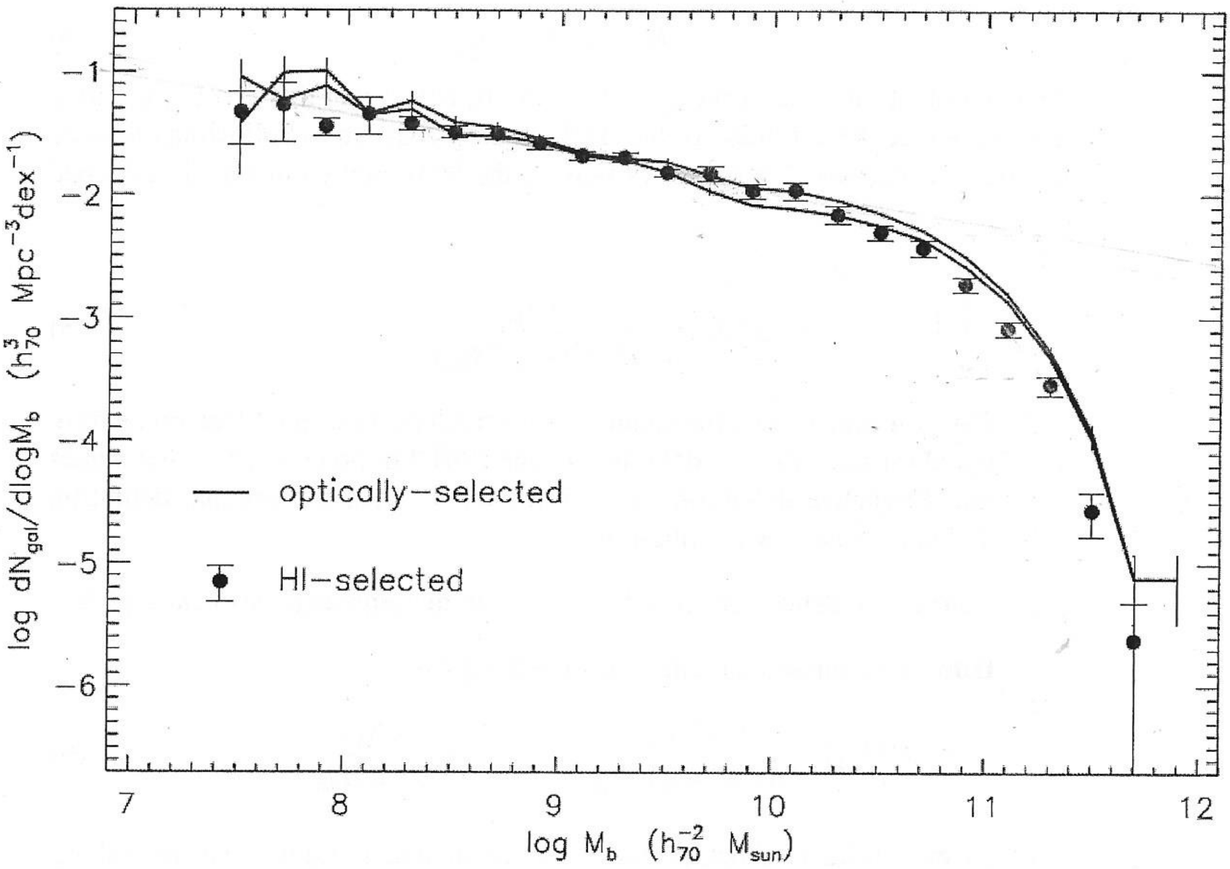

Figure 1: The black dots with errorbars represent the baryonic mass of galaxies (BMF) measured from a combination of data from the SDSS and ALFALFA surveys. The baryonic mass, M_{bar}, is measured in terms of solar masses, $M_{\odot}$. Note that both axes are logarithmic.

In figure 1 you see a plot of the baryonic mass function (BMF) of galaxies. The measurement shown has been obtained from *real data*, in particular from optical data of from the Sloan Digital Sky Survey (SDSS) and the radio data from the ALFALFA survey. (Forget about the two thin lines, only consider the black datapoints and their errorbars). The BMF refers to the "baryonic" mass of galaxies, which in this context is the mass that we can observe with our telescopes. In turn, this usually means the mass of stars (which we can infer from the optical luminosity of a galaxy) plus the mass of hydrogen gas (which we can measure from the radio luminosity of a galaxy):

$$M_{\rm bar} = M_* + M_{\rm gas} \tag{4}$$

Take a look at the axes of the plot. First of all, notice that this is a log-log plot. Second, notice the definition of the BMF on the y-axis label. Following the same naming conventions of variables in Eqn. 1, the BMF plotted in Fig. 1 is defined as:

$$\phi(M_{\rm bar}) = \frac{dN_{\rm gal}}{dV\, d\log_{10}(M_{\rm bar})} \, . \tag{5}$$

f) Can you explain why the authors of this published astronomical article have opted for this different definition of the BMF? In other words, what makes this alternative definition more "informative" than the original definition that you wrote down in question (a)?

g) Convert the Schechter formula of eqn. 1 to the same expression as eqn. 5?

Hint: Your answer should look something like:

$$\phi(M_{\rm bar}) = \frac{dN_{\rm gal}}{dV\, d\log_{10}(M_{\rm bar})} = (\dots) \times \frac{dN_{\rm gal}}{dV\, dM_{\rm bar}} = \dots\dots \tag{6}$$

h) By eye, make a Schechter function fit to the data in figure 1 above. Taking into account the formula you wrote down in question (g), can you approximately deduce the Schechter parameters for this measurement of the BMF? Use your eyes and perhaps a ruler. Sketch on the plot any points or lines that helped you deduce the parameters. Do not forget to write down the parameter values with their units.

Hint: If you remember the typical values of Schechter parameters from lecture, you can double-check if the values that you obtained make sense.

2 the Zel'dovich Formalism

The Zel'dovich approximation is the first-order Lagrangian description of (gravitational) structure evolution. According to the Zel'dovich formalism, the comoving position $\mathbf{x}(t)$ of a particle with Lagrangian (initial) coordinate $\mathbf{q}$, is given by

$$\mathbf{x} = \mathbf{q} + D(t)\,\vec{\psi}(\mathbf{q})\,, \qquad (7)$$

where $\vec{\psi}(\mathbf{q})$ is the displacement vector.

a) What is the expression for the corresponding peculiar velocity $\mathbf{v}$? Write this expression in terms of expansion factor $a(t)$, growth factor $D(t)$, Hubble parameter $H(t)$ and Peebles factor $f(\Omega)$.

b) In the linear regime of clustering, the peculiar velocity $\mathbf{v}$ is linearly proportional to the peculiar gravity $\mathbf{g}$,

$$\mathbf{v} = \frac{2f(\Omega)}{3\Omega H}\mathbf{g}\,. \qquad (8)$$

where

$$\mathbf{g} = -\frac{\nabla\phi}{a}\,. \qquad (9)$$

Use this relation between $\mathbf{v}$ and $\mathbf{g}$ to derive an expression of the displacement field $\vec{\psi}$ and the gravitational potential ϕ.

c) The Poisson equation relates the density perturbation field δ to the gravitational potential ϕ. First specify the expression for the Poisson equaiton in comoving coordinates. Subsequently, infer the relation between the Fourier components $\hat{\delta}(\mathbf{k})$ of the density field and $\hat{\phi}(\mathbf{k})$ of the potential field (in terms of Ω_0 and H_0).

d) Subsequently, derive the expression for the Fourier components $\hat{\vec{\psi}}(\mathbf{k})$ of the displacement field in terms of the density field components $\hat{\delta}(\mathbf{k})$ and write the Fourier expression (integral) for the displacement field $\vec{\psi}(\mathbf{q})$.

e) Given a cosmological power spectrum $P(k)$, how would you set up the initial conditions for an N-body simulation for $N = M^3$ particles whose original Lagrangian positions $\mathbf{q}_i$ ($i = 1, \ldots, N$) are the grid points of an M^3 grid ?

cont'd next page

In the Lagrangian view of structure formation, you follow the deformation of a volume element $d\mathbf{q}$ as matter displaces itself to Eulerian positions $\mathbf{x}$. Hence, mass conservation dictates that the density at a time t evolves according to

$$\rho(\mathbf{x}, t)\, d\mathbf{x} = \rho_u(t)\, d\mathbf{q}\,, \tag{10}$$

where $\rho_u(t)$ is the uniform density at cosmic time t.

f) Explain the relation above, and infer from this the expression for the density perturbation $\delta(\mathbf{x}, t)$ in terms of the determinant of the Jacobian $J(\mathbf{x}, \mathbf{q})$,

$$J(\mathbf{x}, \mathbf{q}) = \left\| \frac{\partial \mathbf{x}}{\partial \mathbf{q}} \right\|, \tag{11}$$

g) Show that the density of a Lagrangian volume according to the Zel'dovich approximation evolves according to

$$\frac{\rho(\mathbf{x}, t)}{\rho_u} = \left\| \frac{\partial \mathbf{x}}{\partial \mathbf{q}} \right\|^{-1} = \frac{1}{[1 - a(t)\lambda_1][1 - a(t)\lambda_2][1 - a(t)\lambda_3]}, \tag{12}$$

where the vertical bars denote the Jacobian determinant, and λ_1, λ_2 and λ_3 are the eigenvalues of the Zel'dovich deformation tensor ψ_{mn},

$$\psi_{mn} = \frac{D(t)}{a(t)} \frac{\partial^2 \Psi}{\partial q_m \partial q_n} = \frac{2}{3a^3 \Omega H^2} \frac{\partial^2 \underline{\phi}}{\partial q_m \partial q_n}. \tag{13}$$

h) Describe how the character of the density evolution expressed in equation 13 implies the emergence of a mass distribution that is dominated by anisotropic structures, ie. the emergence and evolution of the cosmic web. In this description also indicate the sequence of structural evolution, and use sketches.

i) Towards the more advanced stages of evolution, the Zel'dovich approximation starts to break down. Explain what and why this happens.

3 Jeans Instability

We are going to study the development of fluctuations in a medium with pressure.

a) Write down the three equations of motion for a selgravitating collisional medium, ie. a medium with pressure (in physical coordinates). Describe what they express and explain the various terms.

b) Rewrite the equations of motion into comoving coordinates (only write them down, you do not need to derive them), $\mathbf{x} = \mathbf{r}/a(t)$.

c) We are going to assess adiabatic perturbations. What are adiabatic perturbations in terms of the relation between matter and radiation perturbations ? (include a sketch/drawing illustrating the relation between matter and radiation).

d) In addition to adiabatic perturbations we also know isocurvature and isothermal perturbations. Describe these perturbations in terms of relationship matter vs. radiation, and include sketches/drawings in your explanation.

e) For an adiabatic perturbation, show that

$$\nabla p = c_s^2 \nabla \rho \,, \tag{14}$$

where c_s is the adiabatic sound speed,

$$c_s = \left(\frac{\partial p}{\partial \rho}\right)^{1/2} . \tag{15}$$

f) From the equations of motion, derive the second order differential equation for the evolution of the density field δ,

$$\frac{\partial^2 \delta}{\partial t^2} + 2\frac{\dot{a}}{a}\frac{\partial \delta}{\partial t} = \frac{c_s^2}{a^2}\nabla^2 \delta + 4\pi G \rho_u \delta \,. \tag{16}$$

g) Given that in the linear regime each (Fourier) mode $\hat{\delta}(\mathbf{k})$ evolves independently, write the (above) evolution equation for each mode.

cont'd next page

h) From this expression you may infer that perturbations of a size larger than a characteristic size/mass, the Jeans scale, will grow, while smaller scale perturbations will not be able to grow: in the latter case, pressure is able to stop gravitational contraction. Derive the expression for the Jeans mass in terms of sound speed c_s and cosmic density $\rho_u(t)$.

i) In the case of a matter-dominated (early) universe, the universe is expanding like an Einstein-de Sitter universe, $a(t) \propto t^{2/3}$. From the expression in (g) derive the specific (Fourier) expression for evolving perturbations.

j) Give a sketch of the evolution of sub-Jeans and super-Jeans mass perturbations in such a matter-dominated Universe (you do not need to provide or infer the expression for the solution).

k) In the evolving early Universe, both matter and radiation contribute to the density/inertia of the medium. Pressure is almost exclusively provided by radiation. For a plasma of temperature T consisting of radiation with density ρ_r and baryons with density ρ_m, show that

$$c_s^2 = \frac{c^2}{3} \frac{4\rho_r}{4\rho_r + 3\rho_m} \tag{17}$$

l) From the expression above, infer the sound speed in the radiation-dominated Universe at $t \ll t_{eq}$, ie. before the epoch of matter-radiation equivalence.

m) Make a plot of the evolving Jeans mass as a function of time, with clear indication of the equivalence epoch and recombination epoch. Explain and describe the behaviour of the Jeans mass evolution in each of the different regimes. Give approximate mass estimates for the various transition points.

n) What is the definition of the sound horizon at recombination ? Infer an approximate expression in terms of redshift z_{rec} (you may assume an approximate Einstein-de Sitter expansion in this early Universe phase).

o) Make an estimate (in Mpc) of the primordial sound horizon, using the approximation that the sound velocity at recombination is

$$c_s = \frac{c}{\sqrt{3}}. \tag{18}$$

Explain how this can be found back in the detection of Baryonic Acoustic Oscillations.

SUCCES BIJ DIT TENTAMEN !!!!

BEDANKT VOOR JULLIE AANDACHT EN INTERESSE !!!!

Rien